

Consequences of Heavy Quark Symmetries for Hadronic Molecules

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Among the newly observed structures in the heavy quarkonium mass region, some have been proposed to be hadronic molecules. We investigate the consequences of heavy quark flavor symmetry on these heavy meson hadronic molecules. The symmetry allows us to predict new hadronic molecules on one hand, and test the hadronic molecular assumption of the observed structures on the other hand. We explore the consequences of the flavor symmetry assuming the $X(3872)$ and $Z_b(10610)$ as a isoscalar $D\bar{D}^*$ and isovector $B\bar{B}^*$ hadronic molecule, respectively. A series of hadronic molecules composed of heavy mesons are predicted. In particular, there is an isoscalar 1^{++} $B\bar{B}^*$ bound state with a mass about 10580 MeV which may be searched for in the $\Upsilon(1S, 2S)\pi^+\pi^-\pi^0$ mass distribution; the isovector charmonium partners of the $Z_b(10610)$ and the $Z_b(10650)$ are also predicted, which probably corresponds to the very recently observed $Z_c(3900)$ and $Z_c(4025)$ resonances by the BESIII Collaboration.

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Due to ongoing experimental efforts, a series of new heavy quarkonium states, called XYZ states, have been observed in the last decade. Many of them are expected to be of exotic nature, for a comprehensive review we refer to Ref. [1]. Due to the proximity of the masses to certain hadronic thresholds, some of the XYZ states have been proposed to be hadronic molecules, i.e. states that are generated by the interaction between two or more hadrons (they are bound states if they are below the threshold and in the first Riemann sheet, or virtual states and resonances if they are in the second Riemann sheet of the scattering amplitude). For instance, the famous $X(3872)$ discovered by the Belle Collaboration [2] and confirmed by many other experiments was proposed to be a $D\bar{D}^*$ (the charge conjugated particles are implicitly included here and in the following) bound state with an extremely small binding energy [3] or a virtual state [4]; the isovector $Z_b^{\pm,0}(10610)$ and $Z_b^{\pm}(10650)$ resonances reported by the Belle Collaboration [5, 6] have been considered to be $B\bar{B}^*$ and $B^*\bar{B}^*$ hadronic molecules, respectively [7]; the $Y(4660)$ observed in the $\psi'\pi\pi$ mass distribution by the Belle Collaboration [8] and confirmed by BaBar [9] is possibly a $\psi'f_0(980)$ bound state [10]; the $X(4260)$ state observed by the BaBar Collaboration [11] has been suggested to be a $D\bar{D}_1$ molecule [12–14]. Other models also exist for these states. Thus, in order to understand these states and furthermore their binding mechanisms, it is important to find out methods which can distinguish the different scenarios. Decay patterns are often used for this purpose, here we will pursue a different approach using heavy flavor symmetry.

Without developing complicated dynamical models, symmetries are often helpful in describing certain aspects of various physical systems. For a system involving a heavy quark whose mass m_Q is much larger than Λ_{QCD} , flavor and spin symmetries arise by sending m_Q to infinity (for reviews of heavy quark symmetries, see Refs. [15, 16]). Due to spin symmetry, both heavy mesons and heavy quarkonia form spin multiplets, e.g. the $\{D, D^*\}$ and $\{\eta_c, J/\psi\}$. The masses are degenerate in the heavy quark limit, and their interactions with other hadrons are the same at leading order (LO). Heavy quark spin symmetry was already widely used in predicting new hadronic molecules [7, 17–23]. In this work, in addition to spin symmetry, we will argue that heavy quark flavor symmetry is also very useful in the context of hadronic molecules, and may be used to verify hadronic molecular hypothesis and predict new hadronic molecules. As examples, assuming the $X(3872)$ and $Z_b(10610)$ to be the $D\bar{D}^*$ and $B\bar{B}^*$ molecules, we can predict a series of new hadronic molecules, including the $Z_c(3900)$ reported very recently by the BESIII Collaboration [24], later also by Belle [25], and the new $Z_c(4025)$ observed by BESIII [26].

Let us consider the interaction between two heavy hadrons forming a bound state. As far as the hadrons are not too tightly bound, they will not probe the specific details of the interaction binding them at short distances. Moreover, each of the constituent heavy hadrons will be unable to *see* the internal structure of the other heavy hadron. Therefore, we can exploit this separation of scales to formulate an effective field theory (EFT) description of hadronic molecules. Within EFT we express

physical quantities as power series in terms of the ratio Q/M , where Q stands for the momenta of the mesons within a molecule or the pion mass and M is the QCD hadronic mass scale (of the order of the ρ mass or the center of mass momentum necessary for a heavy hadron to excite another). The contribution of physics at the hard scale M is safely encoded in the counterterms of the EFT at low energies [27, 28].

The situation is analogous to that of the EFT formulation of the nucleon-nucleon interaction [29], which we use as a template for the EFT of heavy hadronic molecules. Yet heavy hadrons entail interesting simplifications over nucleons. On the one hand heavy quark symmetry heavily constrains the low-energy interactions among heavy hadrons [19, 30]. On the other hand pion exchanges are in general perturbative [21, 31], in contrast to nuclear physics where they are not [32], and produce small effects. The only exception is the isoscalar bottom sector where the pions might be nonperturbative due to the large masses of bottom mesons [19, 21]. Yet, the inclusion of one pion exchange in this sector only introduces minor modifications of the predictions, c.f. the discussion of the numerical results later on¹. As a consequence, at LO the EFT description only involves energy-independent contact range interactions [21, 22].

We are mainly interested in two manifestations of heavy quark symmetry: heavy quark spin symmetry (HQSS) and heavy flavour symmetry (HFS). Their role can be easily illustrated in the heavy meson-antimeson system with a series of examples. We begin with HQSS as applied to the Z_b and Z'_b , where we assume that they are 1^{+-} $B\bar{B}^*$ and $B^*\bar{B}^*$ isovector molecules, respectively. HQSS implies that the LO non-relativistic isovector heavy meson-antimeson potential is identical in both cases [7, 18, 19]

$$V_{B\bar{B}^*}^{\text{LO}}(1^{+-}) = V_{B^*\bar{B}^*}^{\text{LO}}(1^{+-}), \quad (1)$$

where we have indicated the particle channel in the subscript. This explains why the energy shift of the Z_b/Z'_b states relative to the $B\bar{B}^*/B^*\bar{B}^*$ thresholds is almost the same. For a further example we can consider the $X(3872)$ — the X_c from now on — as a 1^{++} $D\bar{D}^*$ molecule. HQSS then predicts that the potential in the X_c channel is the identical to that of the 2^{++} $D^*\bar{D}^*$ channel [21, 22]:

$$V_{D\bar{D}^*}^{\text{LO}}(1^{++}) = V_{D^*\bar{D}^*}^{\text{LO}}(2^{++}), \quad (2)$$

meaning that we can expect the existence of a 2^{++} HQSS partner of the $X(3872)$. Explicit calculations indicate that its mass should be in the vicinity of 4012 MeV [22]. Following the previous naming pattern, we will call this state the X'_c .

As can be appreciated the exciting feature about heavy meson molecules is their high degree of symmetry. This is even more evident when we consider HFS. According to HFS the interactions involving heavy mesons do not depend on the heavy quark flavour. This means that the heavy meson-antimeson potential is not able to distinguish the D/D^* mesons from the B/B^* ones. If we apply this idea to the X_c , we find

$$V_{D\bar{D}^*}^{\text{LO}}(1^{++}) = V_{B\bar{B}^*}^{\text{LO}}(1^{++}), \quad (3)$$

and the same is true for the potentials in the X'_c , Z_b and Z'_b channels. The consequence of HFS is that heavy meson molecules can appear in flavour multiplets. A resonance in the charm sector might have a counterpart in the bottom sector and vice versa. However, there is a catch. The formation of bound states does not only depend on the strength of the potential, but also on the reduced mass of the two-body system. A higher reduced mass translates into a stronger binding. If the X_c binds, it is more than likely that the X_b — the bottom counterpart of the X_c — binds too. Searching for such a state may even be regarded as a test of the hadronic molecular hypothesis of the $X(3872)$. On the contrary, the shallow nature of the Z_b and Z'_b indicates that their charm counterparts are probably unbound. Yet the Z_c and Z'_c might survive as virtual states or resonances. As we will see, this is indeed the case.

Now we compute the the expected location of the HQSS and HFS partners of the X_c , Z_b and Z'_b . For that, we notice that at LO the EFT potential is simply a contact-range interaction of the type

$$\langle \vec{p} | V_X^{\text{LO}} | \vec{p}' \rangle = C_{0X}, \quad (4)$$

$$\langle \vec{p} | V_Z^{\text{LO}} | \vec{p}' \rangle = C_{1Z}, \quad (5)$$

where the subscripts indicate the isospin and whether we are considering an X - or Z -like channel (see Table I). For finding bound state solutions we iterate this potential in the Lippmann-Schwinger equation (LSE), where the details can be consulted in Ref. [22]. At this point we find it worth commenting that the contact-range potential is singular and requires a regularization and renormalization procedure. We employ a standard gaussian regulator with a cut-off $\Lambda = 0.5 - 1$ GeV, where we have chosen the cut-off window according to the following principles: Λ must be bigger than the wave number of the states, but at the same time must be small enough to preserve heavy quark symmetry and prevent that the theory might become sensitive to the specific details of short distance dynamics. The dependence of results on the cut-off, when it varies within this window, provides an estimate of the expected size of subleading corrections. For a more complete discussion on the choice of the cut-off in nucleon-nucleon systems, see for instance Ref. [33].

For the numerical calculations, we work in the isospin symmetric limit and use the averaged masses of the

¹ Because the isospin factor in the isovector case is only 1/3 of that in the isoscalar case, the pions are perturbative again in the isovector bottom sector.

TABLE I. Various combinations having the same contact term as the $X(3872)$ (left) and $Z_b(10610)$ (right). Here P and P^* represent D, B and D^*, B^* , respectively.

$I(J^{PC})$	C_{0X}	$I(J^{PC})$	C_{1Z}
$0(1^{++})$	$\frac{1}{\sqrt{2}}(P\bar{P}^* - P^*\bar{P})$	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(P\bar{P}^* + P^*\bar{P})$
$0(2^{++})$	$P^*\bar{P}^*$	$1(1^{+-})$	$P^*\bar{P}^*$
$0(2^+)$	D^*B^*	$1(1^+)$	D^*B^*

heavy mesons, which are $M_D = 1867.24$ MeV, $M_{D^*} = 2008.63$ MeV, $M_B = 5279.34$ MeV and $M_{B^*} = 5325.1$ MeV. The value of C_{0X} is determined from reproducing the central value of the Particle Data Group averaged mass of the $X_c(3872)$, 3871.68 ± 0.17 GeV [34]. The resulting value is $C_{0X} = -1.94 \text{ fm}^2$ for $\Lambda = 0.5$ GeV and -0.79 fm^2 for $\Lambda = 1$ GeV [22], where the uncertainties coming from the error in the mass of the X_c are negligible. At this point one may argue that isospin breaking is important for the X_c , owing to its closeness to the $D^0\bar{D}^{0*}$ threshold, but concrete calculations indicate that the effect is tiny for spectroscopy [23]. In turn, the value of C_{1Z} may be fixed using the $Z_b(10610)$ mass. The mass of the $Z_b(10610)$ measured in the $\Upsilon(nS)\pi, h_b(nP)\pi$ distribution 10607.2 ± 2.0 MeV [5] is 1.3σ above the $B\bar{B}^*$ threshold, while the value measured in the $\Upsilon(5S) \rightarrow B\bar{B}^*\pi$ decay 10597 ± 9 MeV [35] overlaps with the $B\bar{B}^*$ threshold. However, these estimations are based on parametrizing the Z_b and Z'_b poles as Breit-Wigner. The analysis of Ref. [36], which overcomes this limitation, suggests that the Z_b and Z'_b are slightly below threshold and have a binding energy of ~ 4.7 MeV and ~ 0.1 MeV respectively. In line with the estimates of Ref. [36], we assume the Z_b binding energy to be 2.0 ± 2.0 MeV, yielding $C_{1Z} = -0.75_{-28}^{+15} \text{ fm}^2$ for $\Lambda = 0.5$ GeV and $-0.30_{-7}^{+3} \text{ fm}^2$ for $\Lambda = 1$ GeV.

With these values we can make predictions by solving the LSE, as previously commented. We summarize our results in Table II. The uncertainties that are listed correspond to taking into account that HQSS and HFS are not exact, but approximate. We expect a Λ_{QCD}/m_Q deviation of the C_{0X} and C_{1Z} value from the heavy quark limit. Taking 300 MeV for Λ_{QCD} [34], and 1.5 GeV and 4.5 GeV for m_c and m_b , respectively, this translates into a relative 20% error in the charm sector and 7% in the bottom one. Actually, the errors are dominated by the uncertainty in the charm sector. When we compute the X_b and X'_b , the relative error of C_{0X} is rather 20% than 7% as its value has been determined from the $X_c(3872)$. We remind that the uncertainties coming from the errors in the mass of the $X_c(3872)$ are negligible in comparison. For the states derived from the Z_b 's we sum the Λ_{QCD}/m_Q and the binding energy errors in quadrature, where the binding error dominates. Some of the states — the partners of the Z_b/Z'_b — are not bound, but virtual.

We indicate this with a “V”.

Among the predicted states, the 2^{++} ones can decay into two heavy pseudoscalar mesons in a D wave, which would introduce a width of order $\mathcal{O}(10 \text{ MeV})$. We refer the refined results taking into account the coupled channels to a forthcoming work. The predicted mass of the $D^*\bar{D}^*$ bound state is higher than the $\chi_{c2}(2P)$ with a mass of 3927.2 ± 2.6 MeV [34], and might be searched for in the same process as the $\chi_{c2}(2P)$, i.e. $\gamma\gamma \rightarrow D\bar{D}$. The data collected at both Belle and BaBar [37, 38] in that range do not have enough statistics for concluding the existence of such a state.

The most robust prediction would be the $B\bar{B}^*$ bound state with $I(J^{PC}) = 0(1^{++})$, to be called $X_b(10580)$, the analogue of the $X_c(3872)$ in the bottom sector. As mentioned earlier, the pions in this sector might be nonperturbative. One may worry about the stability of the results in this sector against including the pions. However, we have checked that one pion exchange only slightly changes the central value of the X_b mass to 10584 MeV and 10567 MeV for $\Lambda = 0.5$ GeV and 1 GeV, respectively (notice that the cut-off dependence decreases). This state should be narrow since the decay into the $B\bar{B}$ is forbidden. It would decay dominantly into a bottomonium and light mesons. Moreover, the difference between the charged and neutral $B\bar{B}^*$ threshold is tiny, and completely negligible when compared with the binding energy. Therefore, unlike the $X_c(3872)$, whose decays exhibit a large isospin breaking, the $X_b(10580)$ would decay into $\Upsilon(nS)\pi\pi\pi$ ($n = 1, 2$) rather than $\Upsilon(nS)\pi\pi$. It can also decay into $\chi_{bJ}(nP)$ and pions. It is worth emphasizing that the existence of such a state is a consequence of HFS and the assumption of the $X_c(3872)$ being a $D\bar{D}^*$ bound state. Searching for it would shed light on the nature of the $X_c(3872)$.

The Z_c and Z'_c appear as virtual states, not very far away from their respective thresholds. However, the uncertainties of the LO calculation are large, of the order of tens of MeV, as indicated by the difference between the results with different cut-off values. From this point of view, the new charged structure $Z_c(3900)$ observed by the BESIII Collaboration [24], and confirmed by the Belle Collaboration [25] and an analysis using the CLEO data [39], is a natural candidate for the partner in the charm sector of the $Z_b(10610)$. Analogously, we expect the recent $Z_c(4025)$ [26] to be the partner of the $Z_b(10650)$. Therefore, we are tempted to identify the $Z_c(3870)$ and $Z_c(4010)$ states reported in Table II with the observed $Z_c(3900)$ and $Z_c(4025)$. We observe that the Z_c and Z'_c are not necessarily virtual: there are subleading order dynamics that can easily move the states above threshold. Most notably at next-to-leading order the EFT potential can develop a short range repulsive barrier. Thus the LO uncertainty also encompasses the possibility that the states might be resonant. There are also corrections coming from coupled channel

TABLE II. Heavy meson-heavy meson combinations having the same contact term as the $X(3872)$ and $Z_b(10610)$, and the predictions of the pole positions, which are understood to correspond to bound states except if we write “V” in parenthesis for denoting a virtual state. When we increase the strength of the potential to account for the various uncertainties, in one case (marked with † in the table) the virtual pole evolves into a bound state. The masses are given in units of MeV.

V_C	$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)	Masses ($\Lambda = 1$ GeV)	Measurements
C_{0X}	$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)	3871.68 (input)	3871.68 ± 0.17 [34]
	$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012_{-5}^{+4}	4012_{-12}^{+5}	?
	$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580_{-8}^{+9}	10539_{-27}^{+25}	?
	$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626_{-9}^{+8}	10584_{-27}^{+25}	?
	$0(2^+)$	D^*B^*	7333.7	7322_{-7}^{+6}	7308_{-20}^{+16}	?
C_{0Z}	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 (input)	10602.4 ± 2.0 (input)	10607.2 ± 2.0 [5] 10597 ± 9 [35]
	$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1	$10648.1_{-2.5}^{+2.1}$	10652.2 ± 1.5 [5] 10649 ± 12 [35]
	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871_{-12}^{+4} (V)	3837_{-35}^{+17} (V)	$3899.0 \pm 3.6 \pm 4.9$ [24] $3894.5 \pm 6.6 \pm 4.5$ [25]
	$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013_{-11}^{+4} (V)	3983_{-32}^{+17} (V)	4026.3 ± 2.6 [26].
	$1(1^+)$	D^*B^*	7333.7	$7333.6_{-4.2}^{\dagger}$ (V)	7328_{-14}^{+5} (V)	?

dynamics, but in general they are at least next-to-next-to-leading order and hence their impact is modest at best. For instance, the Z_c and Z'_c channels couple with each other and with the nearby $h_c(2P)\pi$ and $\psi(2S)\pi$ channels, though in the latter case we do not know the location of these charmonia. Their impact could be enhanced if they are close enough to the Z_c/Z'_c poles (yet they will continue to be subleading). All this indicates that the Z_c and Z'_c are promising candidates to explain the recently observed $Z_c(3900)$ and $Z_c(4025)$ resonances, though further theoretical effort is still required.

To summarize, in this work we have argued that in addition to HQSS, HFS can be used to predict new heavy meson molecules. We have also considered the uncertainties due to the finite mass of the heavy quarks. The predictions are important in understanding the newly observed hadrons in the heavy quarkonium mass region in the sense that, if the XYZ states are hadronic molecules, they will probably have heavy flavour partners that should be searched for. Note that HFS is a symmetry among the coefficients in the interaction of the Lagrangians (or, equivalently, the heavy meson potentials), and not a symmetry in the binding energies (as the kinetic term of the Lagrangian breaks the symmetry): what matters for binding is the potential times the reduced mass of the heavy hadrons. Particularly, we studied in detail the new states that can be derived from the hypothesis that the $X(3872)$ and $Z_b(10610)$ are $D\bar{D}^*$ and $B\bar{B}^*$ hadronic molecules, respectively². Searching for the isoscalar 1^{++} $B\bar{B}^*$ bound state

in the $\Upsilon(1S, 2S)\pi^+\pi^-\pi^0$ channel at hadron colliders or photon-photon collisions would provide valuable information on the structure of the $X(3872)$. In addition, we find promising isovector 1^{+-} $D\bar{D}^*$ and $D^*\bar{D}^*$ virtual states near threshold that could very well be identified with the newly discovered $Z_c(3900)$ [24] and $Z_c(4025)$ [26].

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² We notice that approaches involving phenomenological (i.e.

model-dependent) ingredients – but usually incorporating heavy quark symmetry – can lead to other conclusions: while the X_b is usually predicted, the Z_c is not [20, 40, 41].

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